CALCULUS

FOR EVERYONE

CALCULUS
FOR EVERYONE

UNDERSTANDING
THE MATHEMATICS
OF CHANGE

MITCH STOKES

WITH ILLUSTRATIONS BY SUMMER STOKES

CORRECTION SHEET

ROMAN ROADS PRESS
INHERIT THE HUMANITIES | MOSCOW, IDAHO

8

A NEW TOOL: THE LIMIT

THE ERRORS FOR CHAPTER 8 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

8.4 SAME LIMIT, DIFFERENT BEHAVIOR

Incorrect

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} f(x)$$

limit of g(x) as x approaches 0

value of f(x) approaches 0

Correct

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} f(x)$$

limit of g(x) as x approaches 0

limit of f(x) as x approaches 0

8.8 EXERCISES

Incorrect

f. $\lim_{x\to 3}\left(\frac{x-4}{x^2-x-12}\right)$

g. $\lim_{x\to 4}\left(\frac{x-4}{x^2-x-12}\right)$

h. $\lim_{x\to 1}\left(\frac{(x^2-1)^2}{x-1}\right)$

i. $\lim_{k\to 0}\left(\frac{(x+k)^2-x^2}{k}\right)$

j. $\lim_{x\to 0}\left(\sqrt{x}\right)$

k. $\lim_{x\to 5}\left(\sqrt{x}\right)$

l. $\lim_{x\to 0}\left(\frac{x^3}{x}\right)$

m. $\lim_{x\to 1}\left(\sqrt{x^2-1}\right)$

Correct

f. $\lim_{x\to 3}\left(\frac{x-4}{x^2-x-12}\right)$

g. $\lim_{x\to 4}\left(\frac{x-4}{x^2-x-12}\right)$

h. $\lim_{x\to 1}\left(\frac{(x-1)^2}{x-1}\right)$

i. $\lim_{k\to 0}\left(\frac{(x+k)^2-x^2}{k}\right)$

j. $\lim_{x\to 0}\left(\sqrt{x}\right)$

k. $\lim_{x\to 5}\left(\sqrt{x}\right)$

l. $\lim_{x\to 0}\left(\frac{x^3}{x}\right)$

m. $\lim_{x\to 1}\left(\sqrt{x^2-1}\right)$

9

THE METHOD OF APPROXIMATION AND DEFINING INSTANTANEOUS SPEED

THE ERRORS FOR CHAPTER 9 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

9.2 FREE FALL AND AVERAGE SPEED

Incorrect

We can calculate d_f and d_i as follows:

$$d_i(t_i) = 16t_i^2$$
$$d_f(t_f) = 16t_f^2$$

So given that $t_f = 3$ s and $t_i = 0$ s, as we said, we can write things this way:

$$d_1(1) = 16(1)^2 = 16 \text{ ft}$$
$$d_3(3) = 16(3)^2 = 144 \text{ ft}$$

Correct

We can calculate d_f and d_i as follows:

$$d_i(t_i) = 16t_i^2$$
$$d_f(t_f) = 16t_f^2$$

So given that $t_f = 3$ s and $t_i = 1$ s, as we said, we can write things this way:

$$d_1(1) = 16(1)^2 = 16 \text{ ft}$$
$$d_3(3) = 16(3)^2 = 144 \text{ ft}$$

9.6 STUDY QUESTIONS

Incorrect

Question 11:

For a dropped object the average velocity between 0 and 3 seconds is smaller than the average velocity between the interval covering 2 to 3 seconds. Why is the latter average speed greater even though the interval is smaller?

Correct

Question 11:

For a dropped object the average velocity between 1 and 3 seconds is smaller than the average velocity between the interval covering 2 to 3 seconds. Why is the latter average speed greater even though the interval is smaller?

10

USING THE METHOD OF INCREMENTS TO CALCULATE INSTANTANEOUS SPEED

THE ERROR FOR **SECTION 10.5** WILL BE CORRECTED IN THE NEXT PRINTING OF CALCULUS—**VERSION 1.1.1**

STUDY QUESTION ERRORS HAVE BEEN CORRECTED IN **VERSION 1.1.0**

10.5 APPROACHING THE INSTANT FROM THE OPPOSITE DIRECTION

10.7 STUDY QUESTIONS

Incorrect

Question 7:

In our first example, where did we get $t_i = 3-\Delta t$? How did we get $d_i = 16(3-\Delta t)^2$? How did we find d_f =144 ft/s? Where did we get $t_f = 3$ seconds?

Correct

Question 7:

In our first example, where did we get $t_i = 3-\Delta t$? How did we get $d_i = 16(3-\Delta t)^2$? How did we find d_f =144 ft? Where did we get $t_f = 3$ seconds?

Incorrect

Question 12:

In our second example, where we approached the instant of interest from the opposite direction, what was t_i and how is this different from the t_i in the previous example? What were d_f and t_f? How did we find d_i =144 ft/s? Where did we get $t_i = 3$ seconds?

Correct

Question 12:

In our second example, where we approached the instant of interest from the opposite direction, what was t_i and how is this different from the t_i in the previous example? What were d_f and t_f? How did we find d_i =144 ft? Where did we get $t_i = 3$ seconds?

11

USING THE METHOD OF INCREMENTS TO FIND AN INSTANTANEOUS SPEED FUNCTION

THE ERRORS FOR CHAPTER 11 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

11.1 FINDING *v(t)*

Incorrect

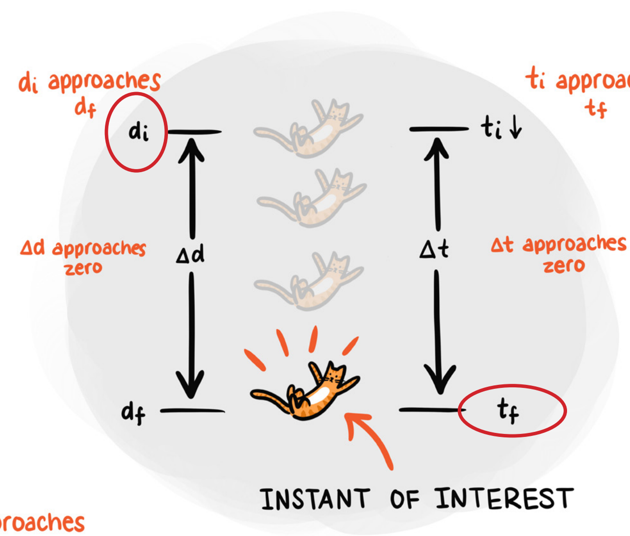


figure 11.1a

Correct

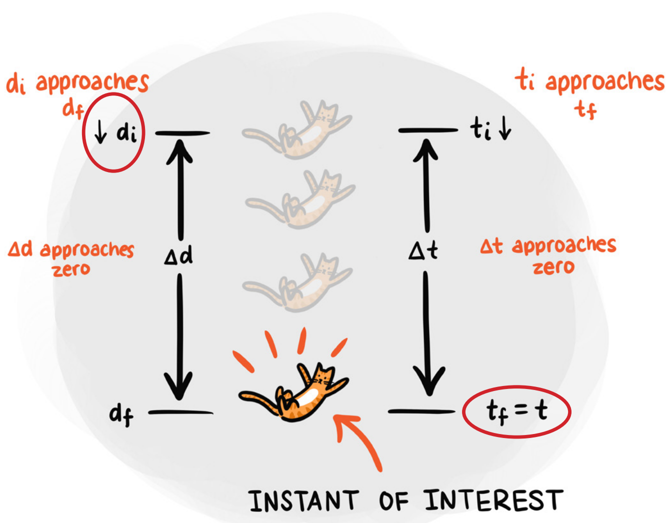


figure 11.1a

Incorrect

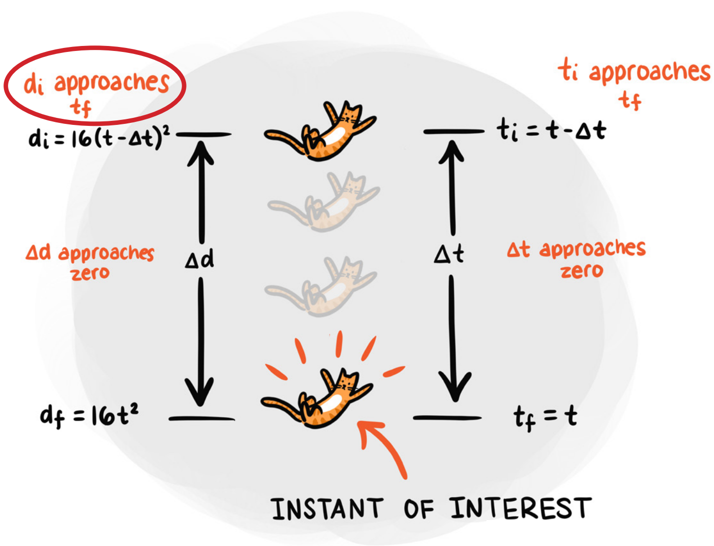


figure 11.1b

Correct

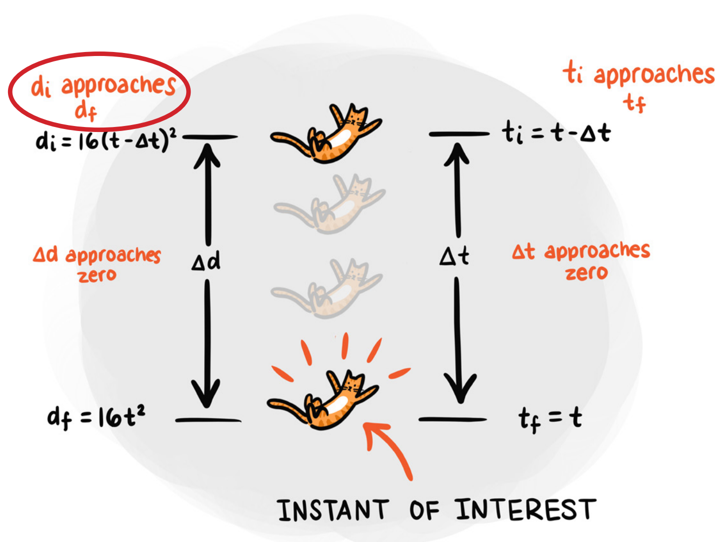


figure 11.1b

Incorrect

$$v = \lim_{\Delta t \to 0} (v_{ave})$$
$$= \lim_{\Delta t \to 0} (32 - 16\Delta t)$$
$$= 32t - 16(0)$$
$$= 32t$$

Correct

$$v = \lim_{\Delta t \to 0} (v_{ave})$$
$$= \lim_{\Delta t \to 0} (32t - 16\Delta t)$$
$$= 32t - 16(0)$$
$$= 32t$$

12

THE DERIVATIVE

THE ERRORS FOR CHAPTER 12 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

12.3 RATE OF CHANGE AT AN INSTANT

Incorrect	Correct
$v_{ave}(t) = \frac{\Delta d}{\Delta t}$ ← the average rate of change in y with respect to time	$v_{ave}(t) = \frac{\Delta d}{\Delta t}$ ← the average rate of change in distance with respect to time

Incorrect	Correct
$v_{ave}(t) = \frac{\Delta x}{\Delta t}$ ← the instantaneous rate of change in x with respect to time	$v_{ave}(t) = \frac{\Delta x}{\Delta t}$ ← the average rate of change in x with respect to time
$v(t) = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\right)$ ← the instantaneous rate of change in x with respect to time	$v(t) = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\right)$ ← the instantaneous rate of change in x with respect to time

12.6 DERIVATIVES VERSUS PLAIN OL' LIMITS

Incorrect	Correct
$\lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x}\right) \neq \lim_{x \to 0} y$	$\lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x}\right) \neq \lim_{x \to 0} y(x)$

13

FINDING MORE DERIVATIVES

THE ERRORS FOR CHAPTER 13 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

13.1 THE DERIVATIVE FOR ANY FUNCTION OF THE FORM *y(x) = ax*²

Incorrect

y evaluated at x — y evaluated at x − Δx

$$\Delta y = a(x)^2 - a(x - \Delta x)^2$$

Correct

y evaluated at x — y evaluated at x − Δx

$$\Delta y = a(x)^2 - a(x - \Delta x)^2$$

13.9 EXERCISES

Incorrect

a. $y(x) = 5x^2$

b. $f(x) = x^2$

c. $h(t) = -16t^2$

d. $g(x) = \frac{15}{\pi} z^2$

Correct

a. $y(x) = 5x^2$

b. $f(x) = x^2$

c. $h(t) = -16t^2$

d. $g(z) = \frac{15}{\pi} z^2$

15

DERIVATIVES AND THE PROBLEM OF CHANGE

THE ERRORS FOR CHAPTER 15 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

15.1 ACCELERATION: HOW FAST SPEED CHANGES

Incorrect

$$d(t) = 16t^2 = kt^n$$
$$d'(t) = v(t) = nkt^{n-1} = 2 \cdot 16x^{2-1} = 2 \cdot 16t = 32t$$

Correct

$$d(t) = 16t^2 = kt^n$$
$$d'(t) = v(t) = nkt^{n-1} = 2 \cdot 16t^{2-1} = 2 \cdot 16t = 32t$$

15.3 DROPPING AN OBJECT

Incorrect

$$h(t) = -16t^2$$

Correct

$$d(t) = -16t^2$$

17

SLOPES AND THE METHOD OF INCREMENTS

THE ERRORS FOR CHAPTER 17 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

17.7 EXERCISES

Incorrect

Exercise 4:

Consider the graph below. Let's call the point we're interested in c and the interval or increment h. Write out the definition of the derivative $f'(x)$ in terms of the limit. Approach c from values *greater than c*. This isn't really new to you, but the form is often how the definition of the derivative is formulated in calculus texts, in terms of c and h.

Correct

Exercise 4:

Consider the graph below. Let's call the x-value we're interested in c and the interval or increment h. Write out the definition of the derivative $f'(x)$ in terms of the limit. Approach c from values *greater than c*. This isn't really new to you, but this form is often how the definition of the derivative is presented in calculus texts, in terms of c and h.

18

SLOPES AND THE PROBLEM OF CHANGE

THE ERRORS FOR CHAPTER 18 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

18.2 SLOPES AND FREE FALL: *d(t)*

Incorrect

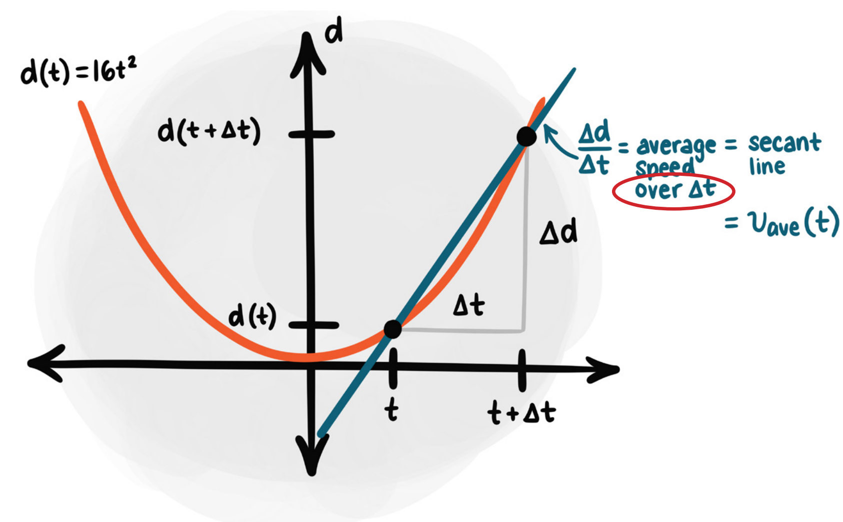


figure 18.2a

Correct

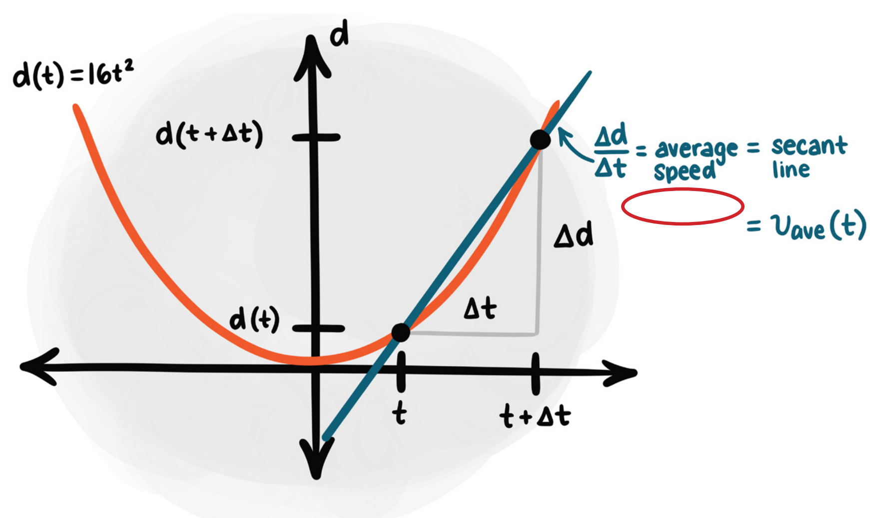


figure 18.2a

19

MORE INFORMATION FROM DERIVATIVES

THE ERRORS FOR CHAPTER 19 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

19.1 EXTRACTING INFORMATION FROM DERIVATIVES

Incorrect

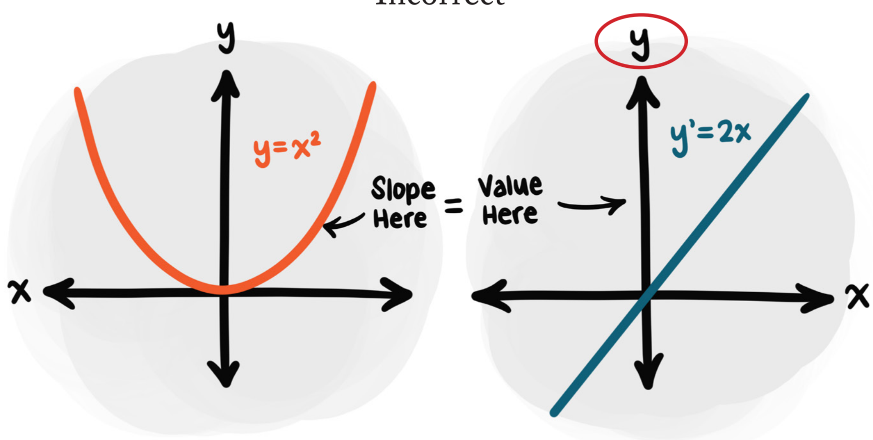


figure 19.1a

Correct

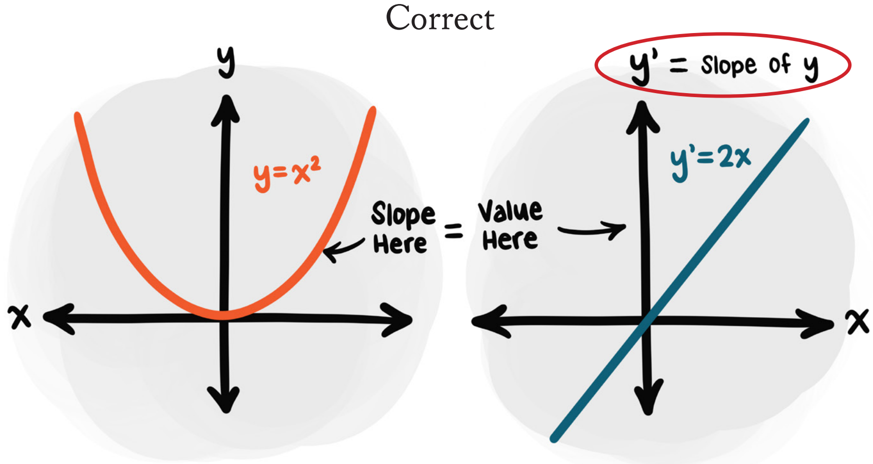


figure 19.1a

Incorrect

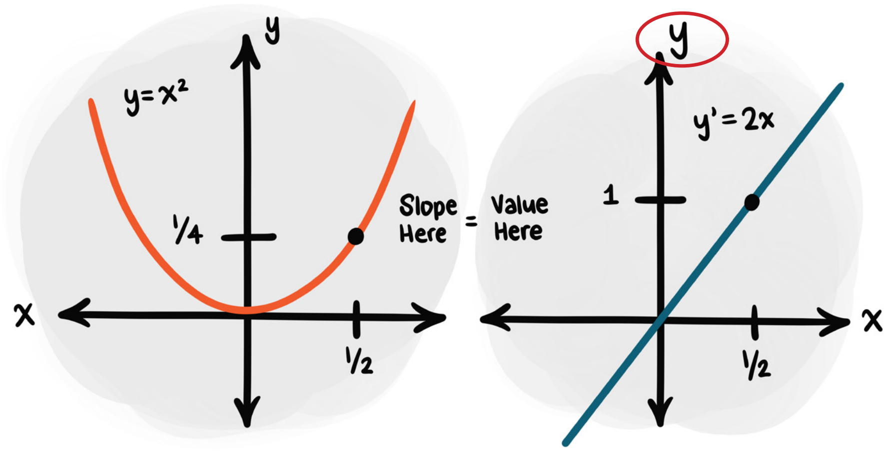


figure 19.1b

Correct

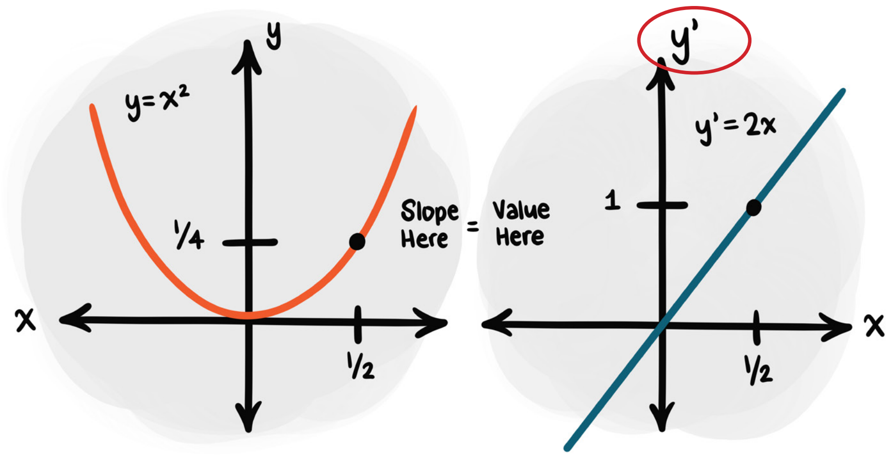
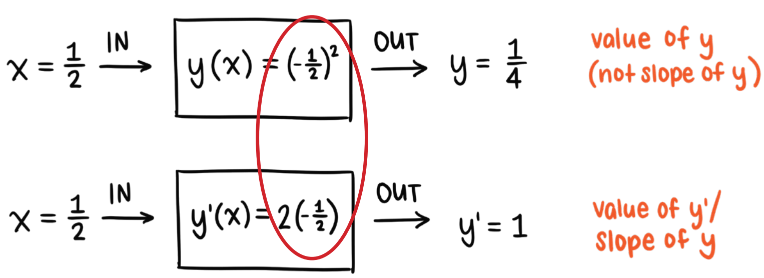
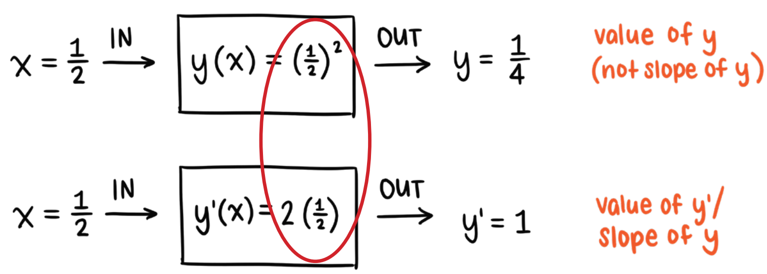


figure 19.1b

Incorrect



Correct



Incorrect

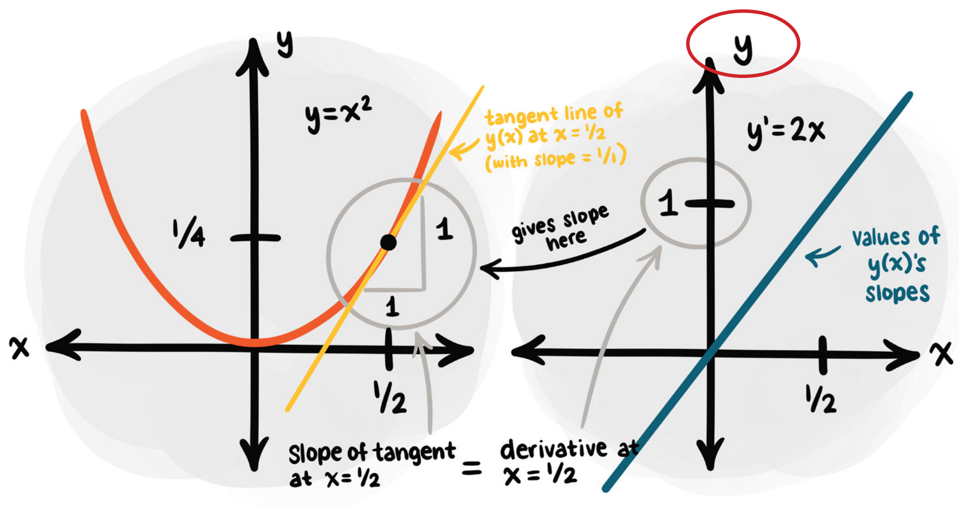


figure 19.1c

Correct

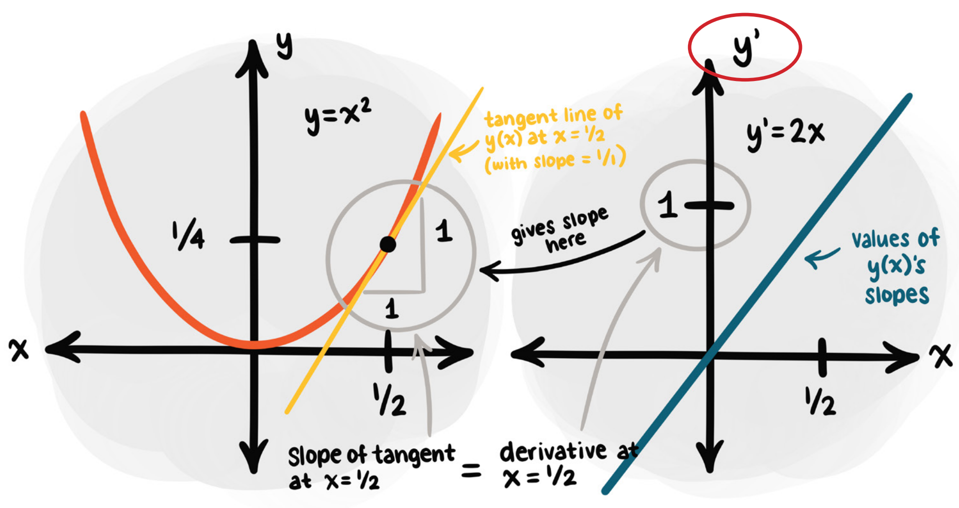


figure 19.1c

Incorrect

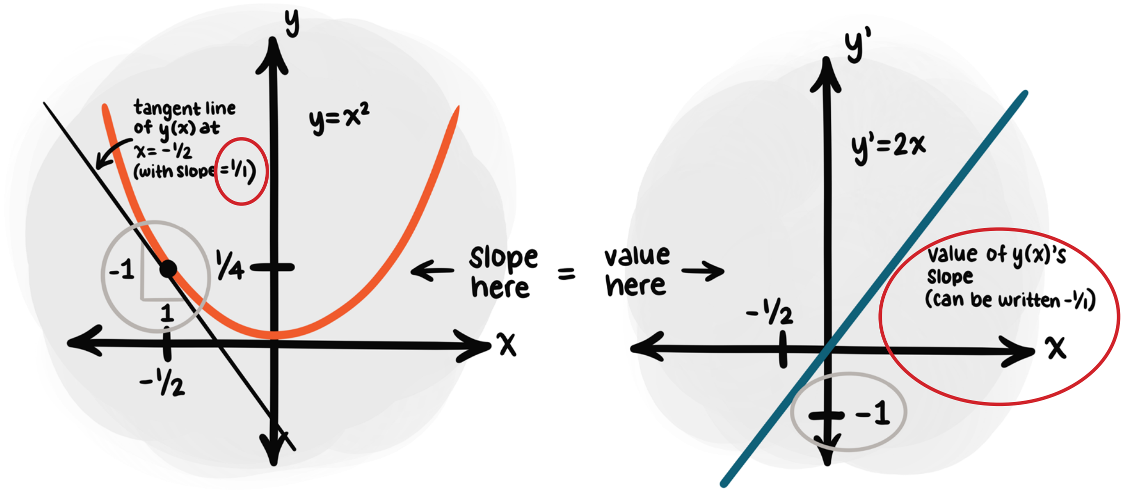


figure 19.1d

Correct

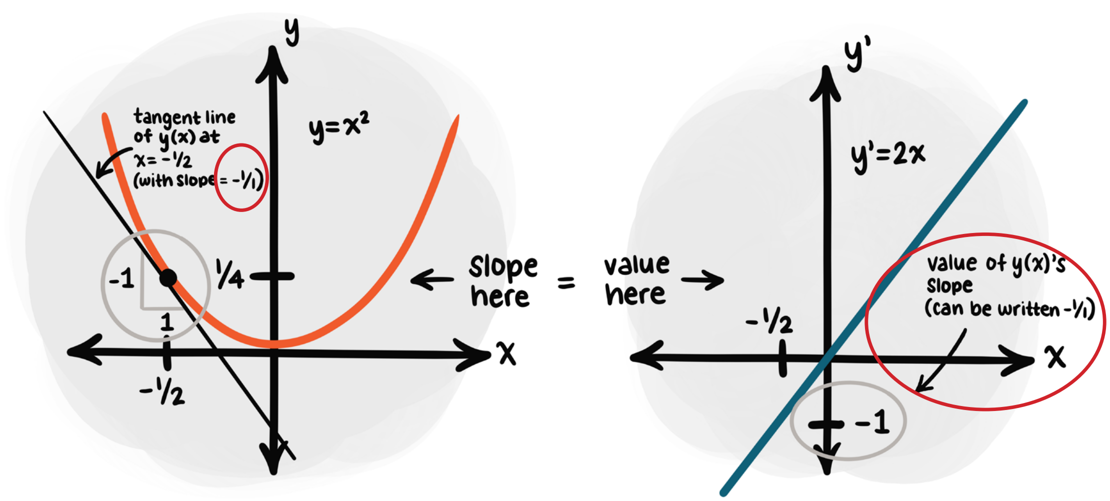
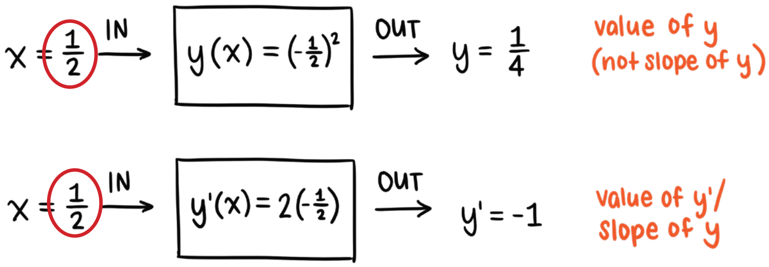
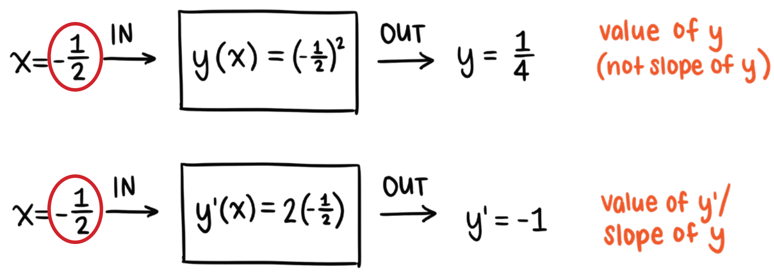


figure 19.1d

Incorrect



Correct



19.5 STUDY QUESTIONS

Incorrect

Question 9:

Write out the two rules for how negative values relate to the steepness of slopes.

Correct

Question 9:

Write out the two rules for how positive and negative values relate to the steepness of slopes.

Incorrect

Question 11:

Draw the graph of $y'''(x) = 0$ by itself. The entire function $y'''(x) = 0$ is flat. What does this say about the function $y'(x) = 2x$?

Correct

Question 11:

Draw the graph of $y'''(x) = 0$ by itself. The entire function $y'''(x) = 0$ is flat. What does this say about the function $y''(x) = 2$?

20

LOOKING CLOSER AT GRAPHS OF FREE FALL

THE ERRORS FOR CHAPTER 20 HAVE BEEN CORRECTED IN **VERSION 1.1.0**
STUDY QUESTION 10 WILL BE CORRECTED IN THE NEXT PRINTING OF CALCULUS—**VERSION 1.1.1**

20.5 INTERPRETING THE VELOCITY FUNCTION'S GRAPH

Incorrect

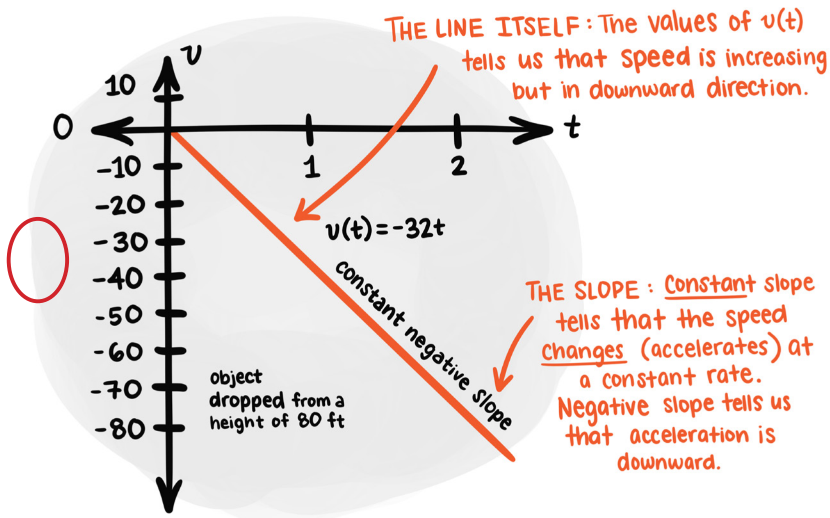


figure 20.5

Correct

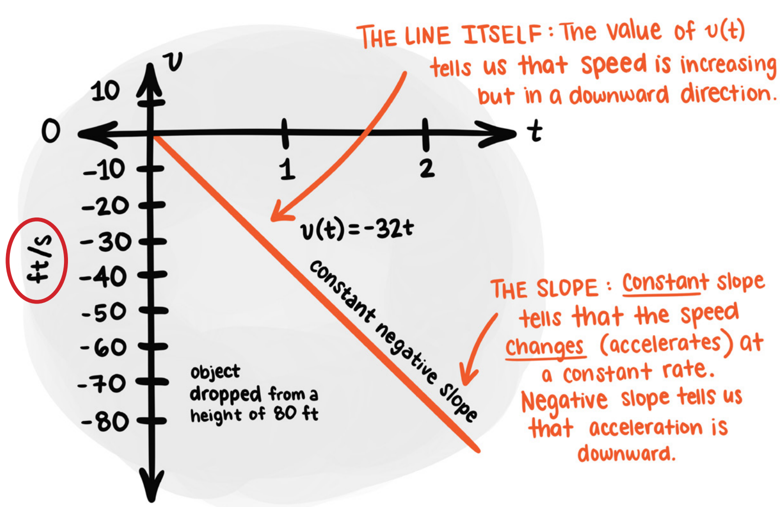


figure 20.5

20.10 STUDY QUESTIONS

Incorrect

Question 10:

Describe the physical situation of the following free fall formula:

$$h(t) = 16t^2 + 80$$

Correct

Question 10:

Describe the physical situation of the following free fall formula:

$$h(t) = -16t^2 + 80$$

Incorrect

Question 18:

For the function

$$h(t) = 16t^2 + 30t + 5$$

Correct

Question 18:

For the function

$$h(t) = -16t^2 + 30t + 5$$

21

THE ANTIDERIVATIVE: UNDOING DERIVATIVES

THE ERRORS FOR CHAPTER 21 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

21.7 THE PROBLEM OF CHANGE AND FINDING C

Incorrect	Correct
$a(t) = -32 = 32t^0$	$a(t) = -32 = -32t^0$

Incorrect

$$h(t) = -16t^2 + 30t + 5$$
$$v(t) = -32t + 30$$
$$a(t) = -32$$

↓ Taking anti-derivatives

Correct

$$h(t) = -16t^2 + 30t + 5$$
$$v(t) = -32t + 30$$
$$a(t) = -32$$

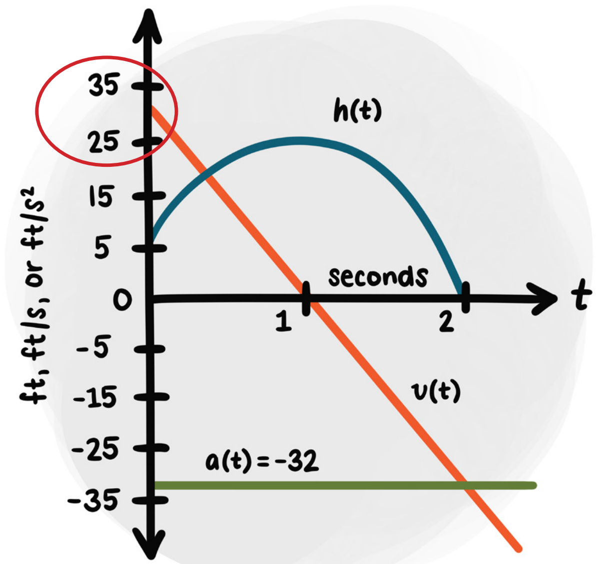
↓ Taking derivatives

21.12 EXERCISES

Incorrect

Exercise 3:

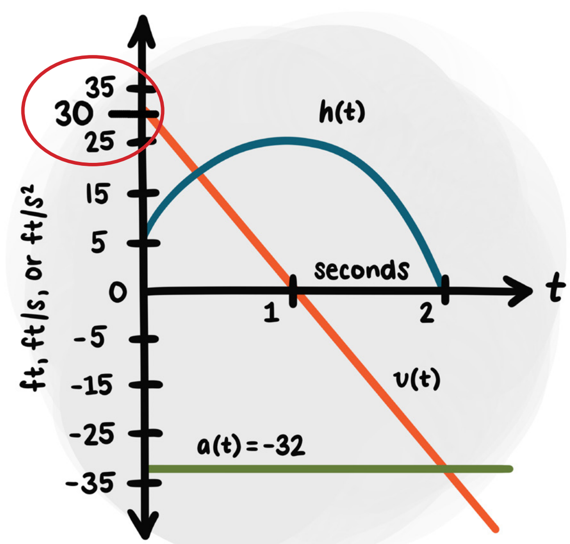
d. An object in free fall whose behavior is described by the following graph:



Correct

Exercise 3:

d. An object in free fall whose behavior is described by the following graph:



23

USING THE METHOD OF SUMMATION TO CALCULATE INTEGRALS

THE ERRORS FOR CHAPTER 21 HAVE BEEN CORRECTED IN **VERSION 1.1.0**

23.3 INSCRIBED AREAS

Incorrect

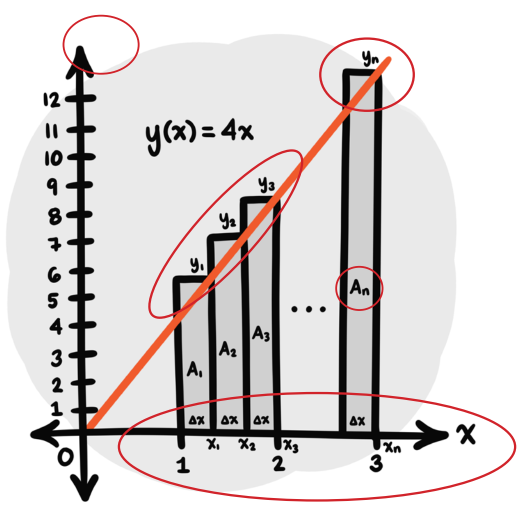


figure 23.3c

Correct

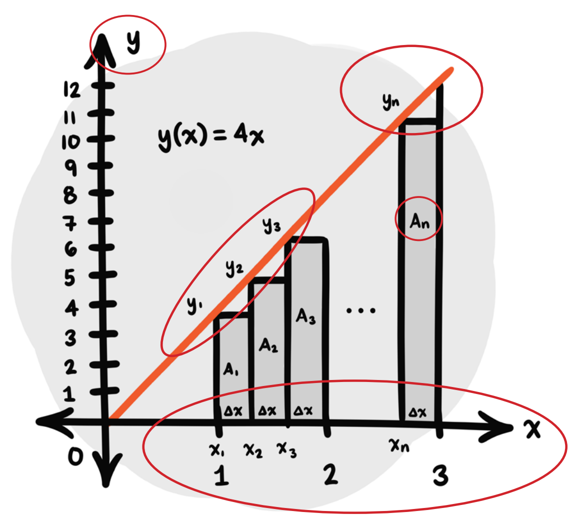


figure 23.3c

Incorrect

$$\underline{S_n} = \underbrace{\overbrace{4}^{y_1}\cdot\Delta x}_{A_1} + \underbrace{\overbrace{4(1+\Delta x)}^{y_2}\cdot\Delta x}_{A_3} + \underbrace{\overbrace{4(1+2\Delta x)}^{y_3}\cdot\Delta x}_{A_3} + \cdots + \underbrace{\overbrace{4[1+(n-1)\Delta x]}^{y_n}\cdot\Delta x}_{A_n}$$

Correct

$$\underline{S_n} = \underbrace{\overbrace{4}^{y_1}\cdot\Delta x}_{A_1} + \underbrace{\overbrace{4(1+\Delta x)}^{y_2}\cdot\Delta x}_{A_2} + \underbrace{\overbrace{4(1+2\Delta x)}^{y_3}\cdot\Delta x}_{A_3} + \cdots + \underbrace{\overbrace{4[1+(n-1)\Delta x]}^{y_n}\cdot\Delta x}_{A_n}$$

Incorrect

$$\underline{S_n} = 4\Delta x + 4\Delta x(1+\Delta x) + 4\Delta x(1+2\Delta x) + \cdots + 4\Delta x[1+(n-1)]$$

Correct

$$\underline{S_n} = 4\Delta x + 4\Delta x(1+\Delta x) + 4\Delta x(1+2\Delta x) + \cdots + 4\Delta x[1+(n-1)\Delta x]$$

Incorrect

$$S_n = 4n\Delta x + 4(\Delta x)^2[1+2+\cdots+(n-1)]$$

Correct

$$\underline{S_n} = 4n\Delta x + 4(\Delta x)^2[1+2+\cdots+(n-1)]$$

Incorrect

$$S_n = 4n\Delta x + 4(\Delta x)^2 \cdot \left[\frac{n(n-1)}{2}\right]$$

Correct

$$\underline{S_n} = 4n\Delta x + 4(\Delta x)^2 \cdot \left[\frac{n(n-1)}{2}\right]$$

23.4 CIRCUMSCRIBED AREAS

Incorrect

$$\overline{S_n} = (y_1 \cdot \Delta x) + (y_2 \cdot \Delta x) + (y_3 \cdot \Delta x) + \cdots + (y_n \cdot \Delta x)$$

$$\overline{S_n} = \underbrace{\overbrace{4(1+\Delta x)}^{y_1}\cdot\Delta x}_{A_1} + \underbrace{\overbrace{4(1+2\Delta x)}^{y_2}\cdot\Delta x}_{A_2} + \underbrace{\overbrace{4(1+\Delta 3)}^{y_3}\cdot\Delta x}_{A_3} \cdots + \underbrace{\overbrace{4(1+n\Delta x)}^{y_n}\cdot\Delta x}_{A_n}$$

$$\overline{S_n} = 4\Delta x(1+\Delta x) + 4\Delta x(1+2\Delta x) + 4\Delta x(1+3\Delta x) + \cdots + 4\Delta x(1+n\Delta x)$$

$$\overline{S_n} = 4\Delta x + 4(\Delta x)^2 + 4\Delta x + 8(\Delta x)^2 + 4\Delta x + 12(\Delta x)^2 + \cdots + 4\Delta x + 4n(\Delta x)^2$$

Correct

$$\overline{S_n} = (y_1 \cdot \Delta x) + (y_2 \cdot \Delta x) + (y_3 \cdot \Delta x) + \cdots + (y_n \cdot \Delta x)$$

$$\overline{S_n} = \underbrace{\overbrace{4(1+\Delta x)}^{y_1}\cdot\Delta x}_{A_1} + \underbrace{\overbrace{4(1+2\Delta x)}^{y_2}\cdot\Delta x}_{A_2} + \underbrace{\overbrace{4(1+3\Delta x)}^{y_3}\cdot\Delta x}_{A_3} \cdots + \underbrace{\overbrace{4(1+n\Delta x)}^{y_n}\cdot\Delta x}_{A_n}$$

$$\overline{S_n} = 4\Delta x(1+\Delta x) + 4\Delta x(1+2\Delta x) + 4\Delta x(1+3\Delta x) + \cdots + 4\Delta x(1+n\Delta x)$$

$$\overline{S_n} = 4\Delta x + 4(\Delta x)^2 + 4\Delta x + 8(\Delta x)^2 + 4\Delta x + 12(\Delta x)^2 + \cdots + 4\Delta x + 4n(\Delta x)^2$$

23.5 MORE (COMPLICATED) EXAMPLES: $y(x) = x^2$

Incorrect

$$\overline{S_n} = (y_1 \cdot \Delta x) + (y_2 \cdot \Delta x) + (y_3 \cdot \Delta x) + \cdots + (y_n \cdot \Delta x)$$
$$\overline{S_n} = \underbrace{\overbrace{(1+\Delta x)^2}^{y_1} \cdot \Delta x}_{A_1} + \underbrace{\overbrace{(1+2\Delta x)^2}^{y_2} \cdot \Delta x}_{A_2} + \underbrace{\overbrace{(1+3\Delta x)^2}^{y_3} \Delta x}_{A_3} + \cdots + \underbrace{\overbrace{(1+n\Delta x)^2}^{y_n} \cdot \Delta x}_{A_n}$$
$$\overline{S_n} = (1+\Delta x)(1+\Delta x)\Delta x + (1+2\Delta x)(1+2\Delta x)\Delta x + (1+3\Delta x)(1+3\Delta x)\Delta x + \cdots + (1+n\Delta x)(1+n\Delta x)\Delta x$$
$$\overline{S_n} = (1+2\Delta x+(\Delta x)^2)\Delta x + (1+4\Delta x+4(\Delta x)^2)\Delta x + (1+6\Delta x+9(\Delta x)^2)\Delta x + \cdots + (1+2n\Delta x+n^2(\Delta x)^2\Delta x$$
$$\overline{S_n} = \Delta x + 2(\Delta x)^2 + (\Delta x)^3 + \Delta x + 4(\Delta x)^2 + 4(\Delta x)^3 + \Delta x + 6(\Delta x)^2 + 9(\Delta x)^3 + \cdots + \Delta x + 2n(\Delta x)^2 + n^2(\Delta x)^3$$

Correct

$$\underline{S_n} = (y_1 \cdot \Delta x) + (y_2 + \Delta x) + (y_3 \cdot \Delta x) + \cdots + (y_n \cdot \Delta x)$$
$$\underline{S_n} = \underbrace{1 \cdot \Delta x}_{A_1} + \underbrace{(1+\Delta x)^2 \Delta x}_{A_2} + \underbrace{(1+2\Delta x)^2 \Delta x}_{A_3} + \cdots + \underbrace{[1+(n-1)\Delta x]^2 \Delta x}_{A_n}$$
$$\underline{S_n} = \Delta x + (1+\Delta x)(1+\Delta x)\Delta x + (1+2\Delta x)(1+2\Delta x)\Delta x + \cdots + [1+(n-1)\Delta x][1+(n-1)\Delta x]\Delta x$$
$$\underline{S_n} = \Delta x + (1+2\Delta x+(\Delta x)^2)\Delta x + (1+4\Delta x+4(\Delta x)^2)\Delta x + \cdots + [1+2(n-1)\Delta x+(n-1)^2(\Delta x)^2]\Delta x$$
$$\underline{S_n} = \Delta x + \Delta x + 2(\Delta x)^2 + (\Delta x)^3 + \Delta x + 4(\Delta x)^2 + 4(\Delta x)^3 + \cdots + [\Delta x + 2(n-1)(\Delta x)^2 + (n-1)^2(\Delta x)^3]$$

Incorrect

$$\overline{S_n} = (y_1 \cdot \Delta x) + (y_2 \cdot \Delta x) + (y_3 \cdot \Delta x) + \cdots + (y_n \cdot \Delta x)$$
$$\overline{S_n} = \underbrace{\overbrace{(1+\Delta x)^2}^{y_1} \cdot \Delta x}_{A_1} + \underbrace{\overbrace{(1+2\Delta x)^2}^{y_2} \cdot \Delta x}_{A_2} + \underbrace{\overbrace{(1+3\Delta x)^2}^{y_3} \Delta x}_{A_3} + \cdots + \underbrace{\overbrace{(1+n\Delta x)^2}^{y_n} \cdot \Delta x}_{A_n}$$
$$\overline{S_n} = (1+\Delta x)(1+\Delta x)\Delta x + (1+2\Delta x)(1+2\Delta x)\Delta x + (1+3\Delta x)(1+3\Delta x)\Delta x + \cdots + (1+n\Delta x)(1+n\Delta x)\Delta x$$
$$\overline{S_n} = (1+2\Delta x+(\Delta x)^2)\Delta x + (1+4\Delta x+4(\Delta x)^2)\Delta x + (1+6\Delta x+9(\Delta x)^2)\Delta x + \cdots + (1+2n\Delta x+n^2(\Delta x)^2\Delta x$$
$$\overline{S_n} = \Delta x + 2(\Delta x)^2 + (\Delta x)^3 + \Delta x + 4(\Delta x)^2 + 4(\Delta x)^3 + \Delta x + 6(\Delta x)^2 + 9(\Delta x)^3 + \cdots + \Delta x + 2n(\Delta x)^2 + n^2(\Delta x)^3$$

Correct

$$\overline{S_n} = (y_1 \cdot \Delta x) + (y_2 \cdot \Delta x) + (y_3 \cdot \Delta x) + \cdots + (y_n \cdot \Delta x)$$
$$\overline{S_n} = \underbrace{\overbrace{(1+\Delta x)^2}^{y_1} \cdot \Delta x}_{A_1} + \underbrace{\overbrace{(1+2\Delta x)^2}^{y_2} \cdot \Delta x}_{A_2} + \underbrace{\overbrace{(1+3\Delta x)^2}^{y_3} \Delta x}_{A_3} + \cdots + \underbrace{\overbrace{(1+n\Delta x)^2}^{y_n} \cdot \Delta x}_{A_n}$$
$$\overline{S_n} = (1+\Delta x)(1+\Delta x)\Delta x + (1+2\Delta x)(1+2\Delta x)\Delta x + (1+3\Delta x)(1+3\Delta x)\Delta x + \cdots + (1+n\Delta x)(1+n\Delta x)\Delta x$$
$$\overline{S_n} = (1+2\Delta x+(\Delta x)^2)\Delta x + (1+4\Delta x+4(\Delta x)^2)\Delta x + (1+6\Delta x+9(\Delta x)^2)\Delta x + \cdots + (1+2n\Delta x+n^2(\Delta x)^2)\Delta x$$
$$\overline{S_n} = \Delta x + 2(\Delta x)^2 + (\Delta x)^3 + \Delta x + 4(\Delta x)^2 + 4(\Delta x)^3 + \Delta x + 6(\Delta x)^2 + 9(\Delta x)^3 + \cdots + \Delta x + 2n(\Delta x)^2 + n^2(\Delta x)^3$$